Keywords: ; ; ; B ; C

Nomenclature: *A*: (^{2}/ ^{3}); C_P: C *P* (/ ^{3}); *D*: B ( ); D_P: D *P* (^{2}/); *EC*: E C (/ ^{3}/); *H*: H ( ); H_P: H *P* ( ); S_{CH_4} : C ; S_{CO_2} : C ; C_{CH_4} : C ; C_{CO_2} : C ;

CH_4

[illegible] [18] [illegible] (H) [19,20] [illegible] (HA) [21,22] [illegible] (AD) [23,24] [illegible] [25,26] [illegible] [27-33].

[illegible] [17] [illegible]

Mathematical Formulation of the Boundary Value Problem

[illegible] [17]. [illegible] CH_4 [illegible] CO_2 [illegible] CH_4 [illegible] CO_2 [illegible] CH_4 [illegible] CO_2 [illegible] [17]:

$$D_{CH_4}\frac{d^2S_{CH_4}}{dx^2} - \sigma\left(S_{CH_4},T\right)=0, \quad (1)$$

$$D_{co2}\frac{d^2S_{co_2}}{dx^2} - \sigma\left(S_{CH_4},T\right)=0, \quad (2)$$

[illegible]:

$$S_{CH_4}(0,z)=\frac{C_{CH_4}(z)}{H_{CH_4}(T)},\ \frac{dS_{CH_4}(\ ,z)}{dx}=0 \text{ for } 0<x<\ ,\ 0<z\le H \quad (3)$$

$$S_{co_2}(0,z)=\frac{C_{co_2}(z)}{H_{co_2}(T)},\ \frac{dS_{co_2}(\ ,z)}{dx}=0 \text{ for } 0<x<\ ,\ 0<z\le H \quad (4)$$

[illegible],

$$\left(S_{CH_4},T\right)=\frac{X_b\,k(T)S_{CH_4}}{K_m+S_{CH_4}};\ H_{CH_4}(T)=\left[4.559(T+273.15)\times10^{(675.74/(T+273.15)-6.880)}\right]^{-1}$$

$$H_{CO_2}(T)=\left[4.559(T+273.15)\times10^{(1012.40/(T+273.15)-6.606)}\right]^{-1};\ K(T)=1.464\times10^{-5}\times1.104^{T-20} \quad (5)$$

[illegible] $CH4$ [illegible] $C\ 2$ [illegible] X_b [illegible] K_m [illegible]

$$S_{CO_2}(z,x) = -B(z)x^2 + 2B(z)x + \frac{C_{CO_2}(z)}{H_{CO_2}(T)} \quad (13)$$

$$B(z) = \frac{-\alpha_{CO_2/CH_4} X_b k(T) \frac{C_{CH_4}(z)}{H_{CH_4}(T)}}{D_{CO_2}\left(K_m + \frac{C_{CH_4}(z)}{H_{CH_4}(T)}\right)} \quad (14)$$

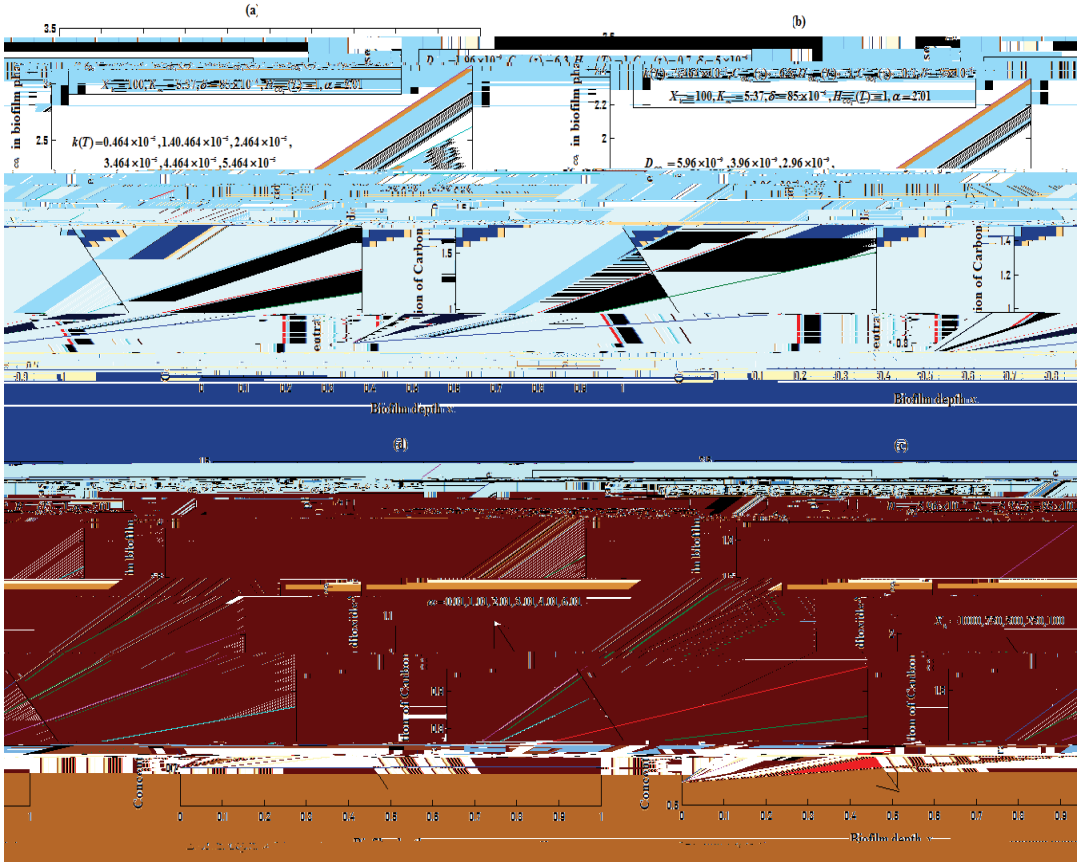
A E . (6-7) (8-9) , CH4 CO2 :

$$C_{CH_4}(z) = C_{CH_{4,in}} \exp\left(\frac{-A\sqrt{D_{CH_4}}\,P\tanh P}{u_g\,H(T)} z\right) \quad (15)$$

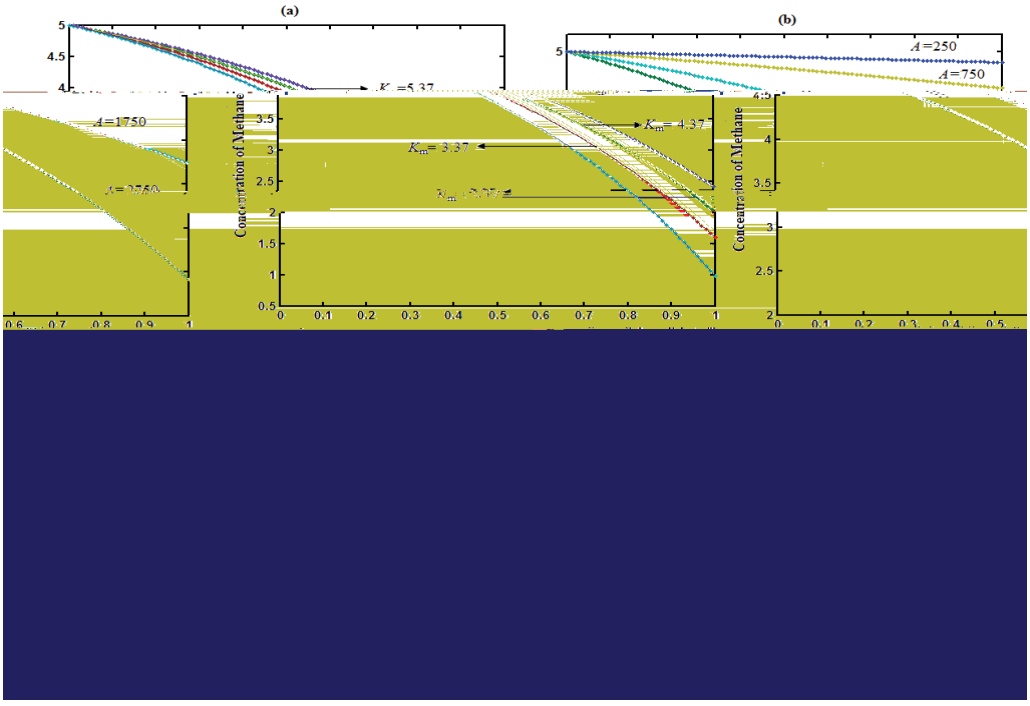
( )
( )
( ) _____

F 3 -3 . F , K m . A (1).

E (15) (16) . F 4 -4 . F F , A K m . A , 5 -5 . F ,



Concentrations of the Carbon dioxide in biofIm phase versus coordinate of biofIm depth for different values of the parameters using Eqn.(13).



Concentrations of the Methane in gas phase methane versus coordinate of height for different values of the parameters using Eqn. (15).

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