Open Access Scientific Reports

Research Article Open Access

Two Sample Median Tests by Ranks

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This paper proposes a two sample median test based on the ranks of sample observations drawn from two independent populations, for testing the null hypothesis of equality of two population medians. The populations may be measurements on, as low as the ordinal scale. It is shown that the proposed test statistic is at least as efficient and powerful as the Mann-Whitney U-Test of the same, over all sample size. When the two samples are of equal size, the proposed test statistic may also be used as an improved alternative to the sign test for two independent samples of equal size. These methods are illustrated with some data, and shown to compare favorably with the ordinary sign-test, the median test and Mann-Whitney U-Test for two independent samples.

Keywords: Mann-Whitney U-Test; Ranks; Two sample; Median; Population

Introduction

e median test is a statistical procedure for testing whether two independent populations di er in their measure of central tendency or location. at is median test enables us determine, whether it is likely that two independent or unrelated samples not necessarily of the same size have been drawn from two populations with equal medians. e median test may be used whenever the observations or scores obtained from the two populations are at least on the ordinal scale of measurement. In these situations, the assumptions of normality and homogeneity necessary for the valid use of the parametric t test may not be satis ed so that parametric tests may not here readily recommend themselves [1].

However, a problem with the median test is that it is based on only the sign or direction of the observations and not on their magnitudes, thereby leading to some loss of information. A procedure that would use both the direction and magnitudes of the observations is likely to be more powerful, and hence, preferable. We propose to develop such a procedure in this paper based on the Kruskal Wallis-One way analysis of variance test by ranks [2].

The Proposed Method

Let x_{ij} be the i^{th} observation in a random sample of size n_j independently drawn from population j for i=1,2,..., n_j; j=1,2.We assume that the two populations are measured on at least the ordinal scale. To apply the two sample median test by ranks, we rst pool the two samples into one combined sample of size $n = \sum_{j=1}^{2} n_j$; $j=1,2 = n_1 + n_2$.

e observations in the pooled sample are now ranked, either from the largest to the smallest, or from the smallest to the largest. Now under the hypothesis of equal population medians, then in the absence of ties, any one randomly selected observation in the combined sample is as likely to be greater as less than any other observation in the sample, and hence, is equally likely to receive any one of the ranks assigned to the observations, thereby justifying the use of the median ranks test for two populations. Let

$$R_j = \sum_{i=1}^{n_j} r_{ij} \quad (1)$$

Be the sum of the ranks assigned to observations drawn from population j for j=1,2, with mean rank

$$\bar{r}_{.j} = \frac{R_j}{n_j} = \sum_{i=1}^{n_j} \frac{r_{ij}}{n_j}$$

e overall mean rank is

$$\bar{r} = \sum_{j=1}^{2} \frac{n_j \bar{r}_{.j}}{n} = \sum_{j=1}^{2}\sum_{i=1}^{n_j} \frac{r_{ij}}{n}$$

at is 6 ... $\frac{n+1}{4}$ ... $\frac{n(n+1)}{12}$

+ + +

=

= + (4)

Now the sum of squared deviations of observed sample or treatment group mean rank $\bar{r}_{.j}$ from their overall mean rank $\bar{r}$ is

$$S^2_{ob} = \sum_{j=1}^{2} n_j(\bar{r}_{.j} - \bar{r})^2 = \sum_{j=1}^{2} \frac{R^2_{.j}}{n_j} - \frac{n(n+1)^2}{4} \quad (5)$$

Now the quadratic form,

$$Q = \frac{S^2_{ob}}{\sigma^2} = \frac{\sum_{j=1}^{2} \frac{R^2_{.j}}{n_j} - \frac{n(n+1)^2}{4}}{\frac{n(n+1)}{12}}$$

at is

$$Q = \frac{12}{n(n+1)} \sum_{j=1}^{2} \frac{R^2_{.j}}{n_j} - 3(n+1)$$

has approximately a chi-square distribution with k-1=2-1=1 degree

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January 16, 2013; February 28, 2013

Oyeka ICA, Okeh UM (2013) Two Sample Median Tests by Ranks. 2: 636 doi:10.4172/scientificreports.636

grade A, assigning it a rank of I, through the lowest grade F assigning it the rank of 25. Tied grades are as usual assigned their mean ranks.

Results

e results are shown in the second and fourth columns of table 1. Using the rank sums shown in table 1 with $n_1=11$ and $n_2=14$ in equation 8, we have

Couple(i)	Husband (x_{i1})	Rank (r_{i1})	Wife (x_{i2})	Rank (r_{i2})	Diff. $d_i = x_{i1} - x_{i2}$	Sign of d_i
1	5	17.5	9	23.5	–4	–
2	0	2.5	3	13	–3	–
3	3	13	2	8.5	1	+
4	3	13	3	13	0	0
5	7	20.5	9	23.5	–2	–
6	0	2.5	0	2.5	0	0
7	8	22	5	17.5	3	+
8	2	8.5	2	8.5	0	0
9	1	5.5	7	20.5	–6	–
10	5	17.5	5	17.5	0	0
11	2	8.5	3	13	–1	–
12	0	2.5	1	5.5	–1	–
Total		133.5 ($R_{.1}$)		166.5		

Family size preferences by a random sample of newly married couples.

n=8 possible + and signs, with a probability of P=0.5 of occurrence, thereby obtaining $P(X \le 2) = \sum_{x=0}^{2} \binom{8}{x} (0.5)^8 = (1+8+28)(0.0039) = 0.1443$.

Since