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Research Article

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Abstract

\$ UHODWLYHO\ QHZ DQDO\WLFDO PHPHPHPHO\WLFDO@p€@WWWUHV€@H\$W
VLP SOH HI¿FLHQW DQG UHOLDEOH ,Q DGGLWLRQ WKH FDOFXODWLRQV LQYROY
GHPRQVWUDWHG WKDW +'0 LV D SRZHUIXO DQG HI¿FLHQW WRRO IRU)UDFWLRQ
GHPRQVWUDWHG WKDW +'0 LV PRUH HI¿FLHQW WKDQ WKH \$'0 \$GRPLDQ GHFRPS
PHWKRG +\$0 +RPRWRS\ DQDO\VLV PHWKRG DQG +30 +RPRWRS\ GHFRPSRVLV

Keywords: Fractional heat-like and wave-like equations; Homotopy decomposition method; Fractional derivative order

Introduction

Fractional Calculus has been used to model physical and engineering processes, which are found to be best described by fractional differential equations. It is worth noting that the standard mathematical models of integer-order derivatives, including nonlinear models, do not work adequately in many cases. In the recent years, fractional calculus has played a very important role in various fields such as mechanics, electricity, chemistry, biology, economics, notably control theory signal, image processing and groundwater problems. In the past several decades, the investigation of travelling-wave solutions for nonlinear equations has played an important role in the study of nonlinear physical phenomena. An excellent literature of this can be found in fractional differentiation and integration operators were also used for extensions of the diffusion and wave equations [1-11].

Exact solutions of Fractional heat-like and wave-like equations with variable coefficients have attracted attention of many authors in mathematics community. Recently, Shou and He [12] used the variational iteration method (VIM) to solve various kinds of heat-like and wave-like equations. However with VIM one needs first to obtain, the Lagrange multiplier and the correctional function. In addition of this, sometime, the solutions obtained via the VIM are noisy [13,14] one therefore needs to cancel the noisy term to obtain the correct solution.

Xu and Cang [15] solved the fractional heat-like and wave-like equations with variable coefficients using Homotopy Analysis Method (HAM). The disadvantage of HAM is that, it is very much depended on choosing auxiliary parameter. Momani [13] applied the Adomian Decomposition method to the time fractional heat-like and wave-like equations with variable coefficients. The main disadvantage of the Adomian method is that the solution procedure for calculation of Adomian polynomials is complex and difficult as pointed out by many researchers [16-20].

In this paper, we extend the application of the Homotopy Decomposition Method (HDM) in order to derive analytical approximate solutions to nonlinear time Fractional heat-like and wave-like equations with variable coefficients

The HDM was recently applied to solve: Fractional modified Kawahara equation, fractional model of HIV infection of CD4+T cells, attractor fractional one-dimensional Keller-Segel equations, fractional Jaulent-Miodek and Whitham-Broer-Kaup equations;

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Basic Idea of the HDM

To illustrate the basic idea of this method we consider a general nonlinear non-homogeneous fractional partial differential equation with initial conditions of the following form

$$\text{—————} ()$$

$$\frac{d}{dt} (x, y) + L \sum_{n=0}^f p^n U_n(x, y) + N \sum_{n=0}^f p^n U_n(x, y) = 0 \quad (17)$$

Comparing the terms of same powers of p gives solutions of various orders with the first term:

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1. RI 'LIIHUHQWLDWLRQ DQG ,QWHJUDWLRQ WR \$UELWUDU\ 2UGHU 'RYHU %RRNV RQ
ODWKHPDWLFV \$FDGHPLF 3UHV 1HZ <RUN 1< 86\$
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 3. .LOEDV \$\$ 6ULYDVWDYD ++ 7UXMLOOR -- 7KHUR\ DQG \$\$SOLFDFWLRQV RI
)UDFWLRQDO 'LIIHUHQWLDQ (TXDWLRQV (OVHYLHU \$PVWHUGDP 7KH 1HWKHUODQG
4. &DSXWR 0 /LQHDU 0RGHOV RI 'LVVLSDFWLRQ ZKR 4 LV DOPRVW)UHTXHQF\
,QGHSHQGHW ,, *HRSK\VLFDQ -RXUQDO ,QWHUQDFWLRQDO